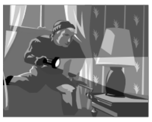
§7 Modelling Uncertainty

- probabilistic uncertainty
  - probability of an outcome
  - dice, shuffled cards
  - statistical reasoning
    - Bayesian networks, Dempster-Shafer theory
- possibilistic uncertainty
  - possibility of classifying object
  - *sorites* paradoxes
  - fuzzy sets

Probabilistic or possibilistic uncertainty?

- Is the vase broken?
- Is the vase broken by a burglar?
- Is there a burglar in the closet?
- Is the burglar in the closet a man?
- Is the man in the closet a burglar?



Bayes' theorem

- hypothesis H
- evidence E
- probability of the hypothesis $P(H)$
- probability of the evidence $P(E)$
- probability of the hypothesis based on the evidence
  $P(H|E) = (P(E|H) \cdot P(H)) / P(E)$

Example

- H — there is a bug in the code
- E — a bug is detected in the test
- $E|H$ — a bug is detected in the test given that there is a bug in the code
- $H|E$ — there is a bug in the code given that a bug is detected in the test

Example (cont'd)

- $P(H) = 0.10$
- $P(E|H) = 0.90$
- $P(E|\neg H) = 0.10$
- $P(E) = P(E|H) \cdot P(H) + P(E|\neg H) \cdot P(\neg H)$
  $= 0.18$
- from Bayes' theorem:
  $P(H|E) = 0.5$
- conclusion: a detected bug has fifty-fifty chance that it is not in the actual code

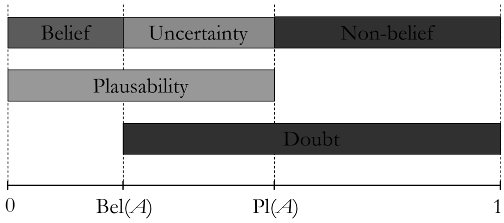
Bayesian networks

- describe cause-and-effect relationships with a directed graph
  - vertices = propositions or variables
  - edges = dependencies as probabilities
- propagation of the probabilities
- problems:
  - relationships between the evidence and hypotheses are known
  - establishing and updating the probabilities

Dempster-Shafer theory

- belief about a proposition as an interval
  [belief, plausability] ⊆ [0, 1]
- belief supporting A: Bel(A)
- plausability of A: Pl(A) = 1 − Bel(¬A)
- Bel(intruder) = 0.3, Pl(intruder) = 0.8
  - Bel(no intruder) = 0.2
  - 0.5 of the probability range is indeterminate

Belief interval



Example 1(5)

- hypotheses: animal, weather, trap, enemy
  - Θ = { A, W, T, E}
- task: assign a belief value for each hypothesis
  - evidence can affect one or more hypotheses
- mass function $m(H)$ = current belief to the set H of hypotheses
  - in the beginning $m(\Theta) = 1$
- evidence 'noise' supports A, W and E
  - mass function $m_n(\{ A, W, E \}) = 0.6$, $m_n(\Theta) = 0.4$

Example 2(3)

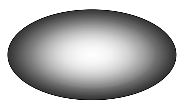
- evidence 'footprints' supports A, T, E
  - $m_f(\{ A, T, E \}) = 0.8$, $m_f(\Theta) = 0.2$
- combination with Dempster's rule:
  - $m_{nf}(\{A, E\}) = 0.48$, $m_{nf}(\{W, A, E\}) = 0.12$, $m_{nf}(\{A, T, E\}) = 0.32$, $m_{nf}(\Theta) = 0.08$
- enemy, trap, trap or enemy, weather, or animal?
  - Bel(E) = 0, Pl(E) = 1
  - Bel(T) = 0, Pl(T) = 0.4
  - Bel(T, E) = 0, Pl(T, E) = 1
  - Bel(W) = 0, Pl(W) = 0.2
  - Bel(A) = 0, Pl(A) = 1

Example 3(3)

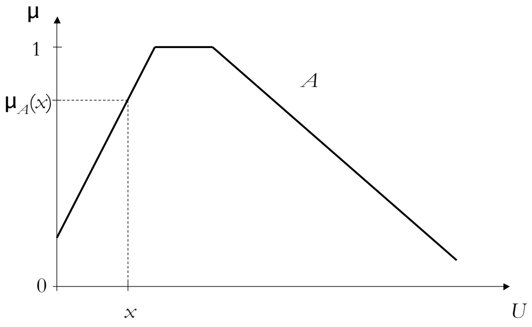
- evidence 'candy wrapper' supports T, E
  - $m_c(\{E\}) = 0.6$, $m_c(\{T\}) = 0.3$, $m_c(\Theta) = 0.1$
- combination with Dempster's rule:
  - $m_{nfc}(\{E\}) = 0.73$, $m_{nfc}(\{T\}) = 0.15$, $m_{nfc}(\{A, E\}) = 0.06$, $m_{nfc}(\{A, T, E\}) = 0.04$, $m_{nfc}(\{W, A, E\}) = 0.01$, $m_{nfc}(\Theta) = 0.01$
- enemy, trap, trap or enemy, weather, or animal?
  - Bel(E) = 0.73, Pl(E) = 0.85
  - Bel(T) = 0.15, Pl(T) = 0.2
  - Bel(T, E) = 0.88, Pl(T, E) = 1
  - Bel(W) = 0, Pl(W) = 0.02
  - Bel(A) = 0, Pl(A) = 0.03

Fuzzy sets

- element x has a membership in the set A defined by a membership function $\mu_A(x)$
  - not in the set: $\mu_A(x) = 0$
  - fully in the set: $\mu_A(x) = 1$
  - partially in the set: $0 < \mu_A(x) < 1$
- contrast to classical 'crisp' sets
  - not in the set: $\chi_A(x) = 0$
  - in the set: $\chi_A(x) = 1$



Membership function



How to assign membership functions?

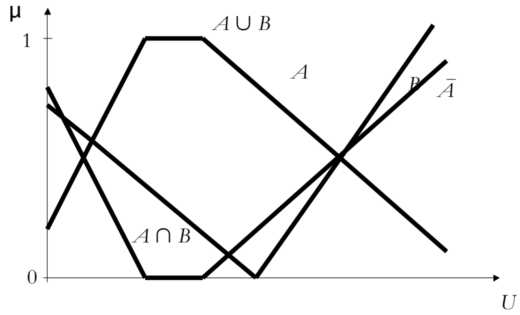
- real-word data
  - physical measurements
  - statistical data
- subjective evaluation
  - human experts' cognitive knowledge
  - questionnaires, psychological tests
- adaptation
  - neural networks, genetic algorithms
- → simple functions usually work well enough as long as they model the general trend

Fuzzy operations

- union: $\mu_C(x) = \max\{\mu_A(x), \mu_B(x)\}$
- intersection: $\mu_C(x) = \min\{\mu_A(x), \mu_B(x)\}$
- complement: $\mu_C(x) = 1 - \mu_A(x)$

- note: operations can be defined differently

Fuzzy operations (cont'd)



Uses for fuzzy sets

- approximate reasoning
- fuzzy constraint satisfaction problem
- fuzzy numbers
- almost any 'crisp' method can be fuzzified!