

# Algorithms and Networking for Computer Games

## Chapter 7: Modelling Uncertainty

# Types of uncertainty

- probabilistic uncertainty
  - probability of an outcome
  - dice, shuffled cards
  - statistical reasoning
    - Bayesian networks, Dempster-Shafer theory
- possibilistic uncertainty
  - possibility of classifying object
  - *sorites* paradoxes
  - fuzzy sets

# Bayes' theorem

- hypothesis  $H$
- evidence  $E$
- probability of the hypothesis  $P(H)$
- probability of the evidence  $P(E)$
- probability of the hypothesis based on the evidence

$$P(H|E) = (P(E|H) \cdot P(H)) / P(E)$$

# Example

- $H$  — there is a bug in the code
- $E$  — a bug is detected in the test
- $E|H$  — a bug is detected in the test given that there is a bug in the code
- $H|E$  — there is a bug in the code given that a bug is detected in the test

# Example (cont'd)

- $P(H) = 0.10$
- $P(E | H) = 0.90$
- $P(E | \neg H) = 0.10$
- $P(E) = P(E | H) \cdot P(H) + P(E | \neg H) \cdot P(\neg H)$   
 $= 0.18$
- from Bayes' theorem:  
 $P(H | E) = 0.5$
- conclusion: a detected bug has fifty-fifty chance that it is not in the actual code

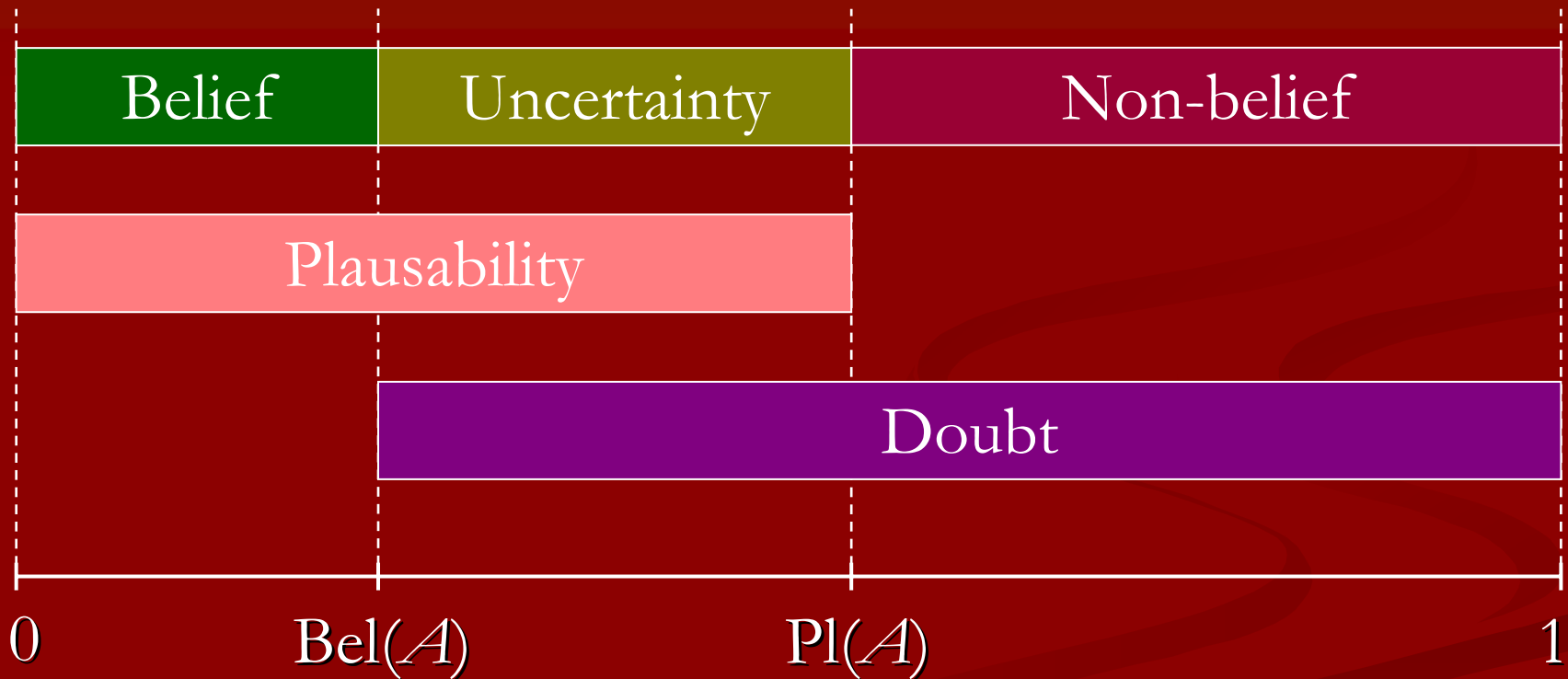
# Bayesian networks

- describe cause-and-effect relationships with a directed graph
  - vertices = propositions or variables
  - edges = dependencies as probabilities
- propagation of the probabilities
- problems:
  - relationships between the evidence and hypotheses are known
  - establishing and updating the probabilities

# Dempster-Shafer theory

- belief about a proposition as an interval  
[ belief, plausability ]  $\subseteq$  [ 0, 1 ]
- belief supporting  $A$ :  $\text{Bel}(A)$
- plausability of  $A$ :  $\text{Pl}(A) = 1 - \text{Bel}(\neg A)$
- $\text{Bel}(\text{intruder}) = 0.3$ ,  $\text{Pl}(\text{intruder}) = 0.8$ 
  - $\text{Bel}(\text{no intruder}) = 0.2$
  - 0.5 of the probability range is indeterminate

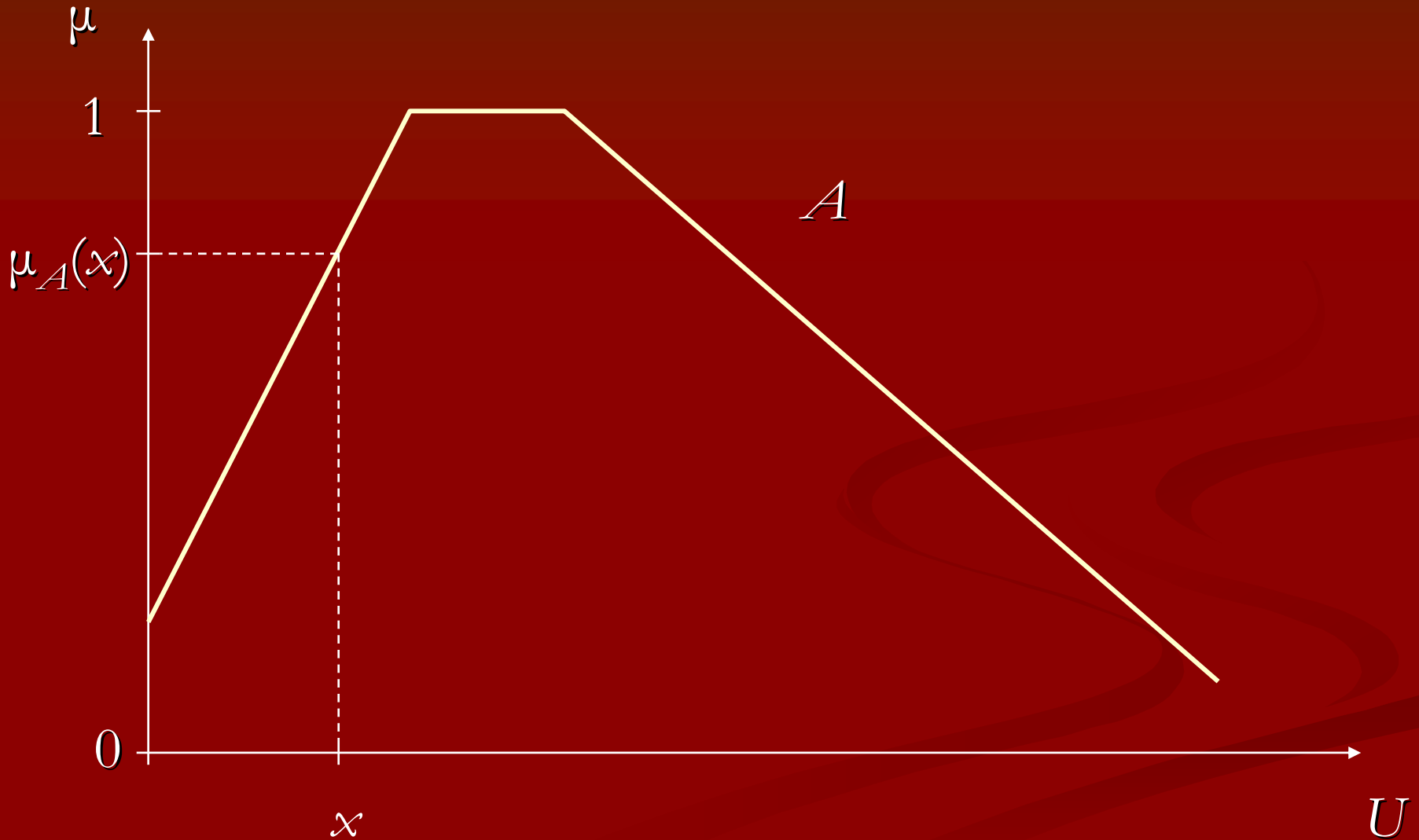
# Belief interval



# Fuzzy sets

- element  $x$  has a membership in the set  $A$  defined by a membership function  $\mu_A(x)$ 
  - not in the set:  $\mu_A(x) = 0$
  - fully in the set:  $\mu_A(x) = 1$
  - partially in the set:  $0 < \mu_A(x) < 1$

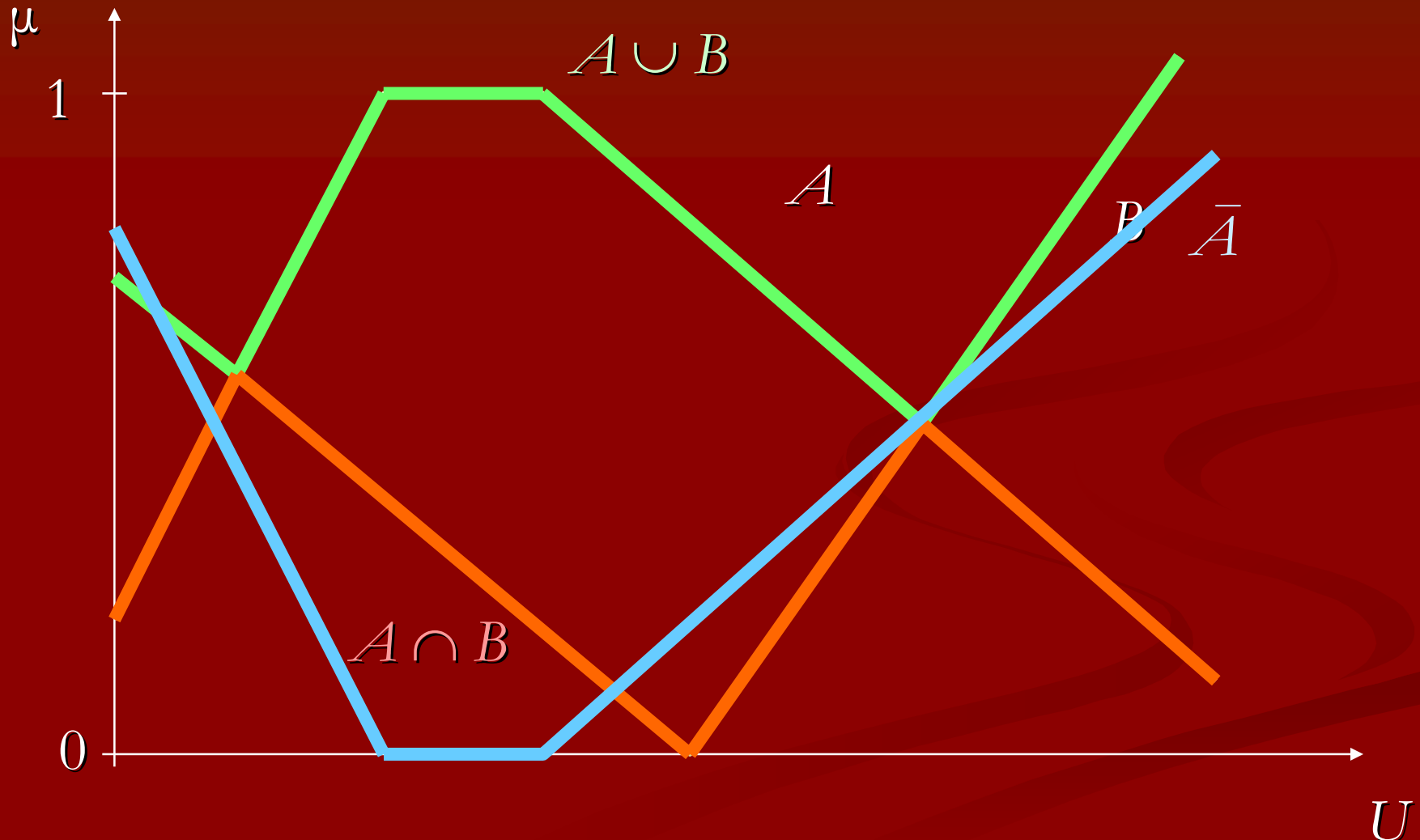
# Membership function



# Fuzzy operations

- union:  $\mu_C(x) = \max\{\mu_A(x), \mu_B(x)\}$
- intersection:  $\mu_C(x) = \min\{\mu_A(x), \mu_B(x)\}$
- complement:  $\mu_C(x) = 1 - \mu_A(x)$
  
- note: operations can be defined differently

# Fuzzy operations (cont'd)



# Uses for fuzzy sets

- approximate reasoning
- fuzzy constraint satisfaction problem
- fuzzy numbers
- almost any ‘crisp’ method can be fuzzified!