## Evaluation function

- expand vertex minimizing

  $$f(v) = g(s \sim> v) + h(v \sim> r)$$
- $g(s \sim> v)$ estimates the minimum cost from the start vertex to $v$
- $h(v \sim> r)$ estimates (heuristically) the cost from $v$ to the goal vertex
- if we had exact evaluation function $f^*$, we could solve the problem without expanding any unnecessary vertices

## Cost function *g*

- actual cost from $s$ to $v$ along the cheapest path found so far
  - exact cost if $G$ is a tree
  - can never underestimate the cost if $G$ is a general graph
- $f(v) = g(s \sim> v)$ and unit cost $\longrightarrow$ breadth-first search
- $f(v) = -g(s \sim> v)$ and unit cost $\longrightarrow$ depth-first search

## Heuristic function *h*

- carries information from outside the graph
- defined for the problem domain
- the closer to the actual cost, the less superfluous vertices are expanded
- $f(v) = g(s \sim> v) \longrightarrow$ cheapest-first search
- $f(v) = h(v \sim> r) \longrightarrow$ best-first search

## Admissibility

- let Algorithm A be a best-first search using the evaluation function $f$
- search algorithm is *admissible* if it finds the minimal path (if it exists)
  - if $f = f^*$, Algorithm A is admissible
- Algorithm A* = Algorithm A using an estimate function $h$
  - A* is admissible, if $h$ does not overestimate the actual cost


## Monotonicity

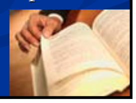
- $h$ is locally admissible $\longrightarrow$ $h$ is monotonic
- monotonic heuristic is also admissible
- actual cost is never less than the heuristic cost $\longrightarrow f$ will never decrease
- monotonicity $\longrightarrow$ A* finds the shortest path to any vertex the first time it is expanded
  - if a vertex is rediscovered, path will not be shorter
  - simplifies implementation

## Optimality

- Optimality theorem: The first path from $s$ to $r$ found by A* is optimal.
- Proof: lecture notes pp. 94-95

## Informedness

- the more closely *h* approximates $h^*$, the better A* performs
- if $A_1$ using $h_1$ will never expand a vertex that is not also expanded by $A_2$ using $h_2$, $A_1$ is more informed that $A_2$
- informedness → no other search strategy with *the same amount of outside knowledge* can do less work than A* and be sure of finding the optimal solution



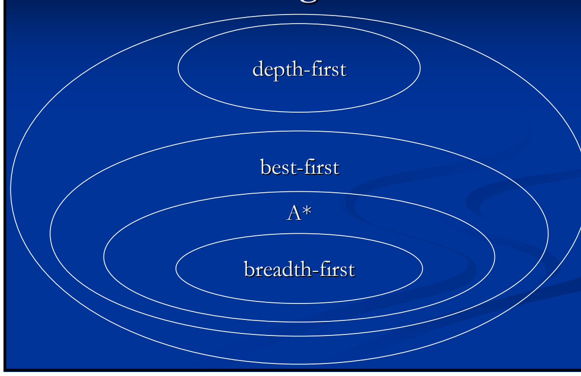
## Algorithm A*

- because of monotonicity
  - all weights must be positive
  - closed list can be omitted
- the path is constructed from the mapping π starting from the goal vertex
  - $s \rightarrow \ldots \rightarrow \pi(\pi(\pi(r))) \rightarrow \pi(\pi(r)) \rightarrow \pi(r) \rightarrow r$

## Practical considerations

- computing *h*
  - despite the extra vertices expanded, less informed *h* may yield computationally less intensive implementation
- suboptimal solutions
  - by allowing overestimation A* becomes inadmissible, but the results may be good enough for practical purposes

## Search algorithms




## Realizing the movement

- movement through the waypoints
  - unrealistic: does not follow the game world geometry
  - aesthetically displeasing: straight lines and sharp turns
- improvements
  - line-of-sight testing
  - obstacle avoidance
- combining path finding to user-interface
  - real-time response

## Recapitulation

1. discretization of the game world
   - grid, navigation mesh
   - waypoints, connections, costs
2. path finding in a graph
   - Algorithm A*
3. realizing the movement
   - geometric corrections
   - aesthetic improvements

## Alternatives?

- Although this is the *de facto* approach in (commercial) computer games, are there alternatives?

- possible answers
  - AI processors (unrealistic?)
  - robotics: reactive agents (unintelligent?)
  - analytical approaches (inaccessible?)